

**The Effect of Viscosity on Coefficients of Heat
Transfer in Forced Circulation
Evaporators**

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THE EFFECT OF VISCOSITY ON COEFFICIENTS OF HEAT TRANSFER IN FORCED-CIRCULATION EVAPORATORS

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It has been recognized since the beginning of work on heat-transfer coefficients in evaporators that viscosity played an important part. None of the previous investigators studied viscosity at any length.

Claassen^{2,3} boiled sugar solutions of different concentrations and showed that heat-transfer coefficients decreased as the concentration (and therefore the viscosity) increased. Walker, Lewis, and McAdams⁵ suggested that the change in heat-transfer coefficients with boiling point in a natural-circulation evaporator was due to change in viscosity; and suggested that if the apparent coefficient were plotted against the product of viscosity and apparent temperature drop, a straight line should result that was independent of boiling point. Badger¹ has shown that this is not true, although such a method of plotting does account for part of the changes in coefficients with changes in boiling points. However, all this work was carried out in natural-circulation evaporators and is inconclusive; because a change in viscosity causes a change in velocity of circulation, and it is difficult to reach positive conclusions regarding the effect of viscosity alone.

In studying the effect of viscosity on heat transfer coefficients in evaporators, it seemed that the forced-circulation evaporator offered many advantages for preliminary studies of this type. Its principle advantage is that the velocity can be controlled irrespective of viscosity, and therefore the effect of viscosity could be more easily evaluated.

Such work as that of Logan, Fragen and Badger⁴ has shown that when the mean temperature of liquid from top to bottom of the evaporator tube is known, the liquid film coefficient in a forced-circulation

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evaporator can be calculated from the Dittus and Boelter equation with a slight change in constant. This does not help the designer, however, because up to the present no method has been found for predicting the mean temperature of the liquid when the major operating conditions of the evaporator have been established. It is highly desirable to have the effect of viscosity or other factors expressed in terms of the apparent boiling point, the apparent temperature drop, and the apparent overall coefficients, since these are the quantities the designer has available.

It is obvious that when accurate information is to be obtained, the individual film coefficients must be determined. However, when the viscosity of the liquid is high, the liquid-film coefficient is low and the overall coefficient is so nearly equal to the liquid-film coefficient that it is little affected by any possible changes in the steam-film coefficient. It is possible to confine such a study to such values of the liquid-film coefficient as are sensibly equal to the overall coefficient, and the experimental difficulties involved in a determination of metal temperatures are eliminated.

The scope of this paper, therefore, is a study of the effect of viscosity on apparent overall heat-transfer coefficients in a forced-circulation evaporator, under such conditions that the liquid-film coefficients are sensibly equal to the overall coefficient. A further condition is that the results are to be expressed in terms of temperatures available to the designer.

APPARATUS AND EXPERIMENTAL PROCEDURE

The experimental unit was a Swenson semi-commercial forced-circulation evaporator with a heating surface on the liquid side of 20.3 square feet. The tubes were copper, $\frac{7}{8}$ in. O.D., 16 B.W.G. (0.065 in.) wall thickness, 8 ft. long. The liquid was circulated by a Kinney positive pressure pump. A diagrammatic flow sheet of the evaporator and its accessories is given in Fig. 1, which is self-explanatory.

The liquid used in most of the work was molasses solution of different concentrations, to which a small amount of mercuric chloride was added as preservative. A run was started with a solution of given concentration, and as evaporation took place, distilled water was fed to keep the level constant. Thus the concentration of the solution within the evaporator did not vary appreciably during the run; but as checks on this, samples were taken at both beginning and end. Con-

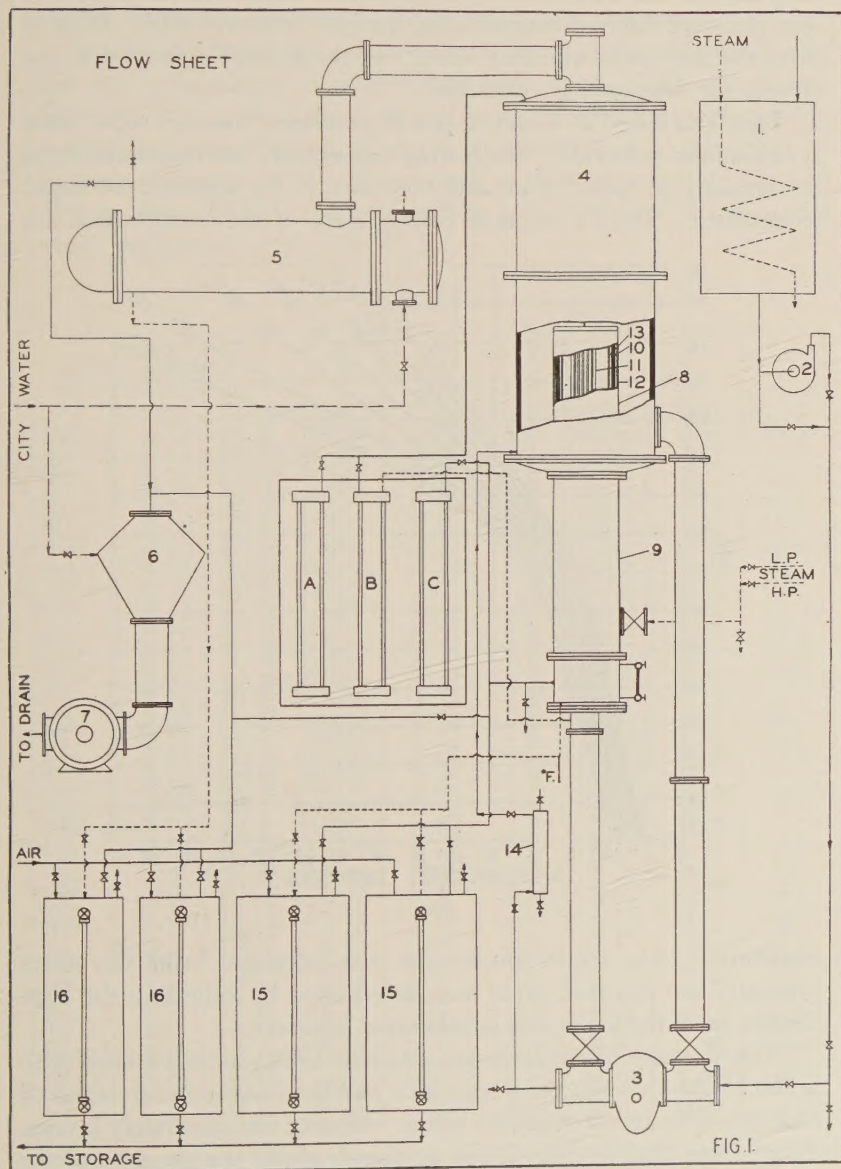


FIG. 1.

centrations were determined by a Brix hydrometer. If the initial and final densities differed considerably, the runs were rejected. In most cases the difference was very small and the average was used as the density for determining viscosities.

Each run lasted 20 minutes, and all necessary readings were taken at two-minute intervals. The boiling temperature was calculated from the pressure in vapor space and therefore is the apparent or vapor temperature. The elevation of boiling point of the solution was not

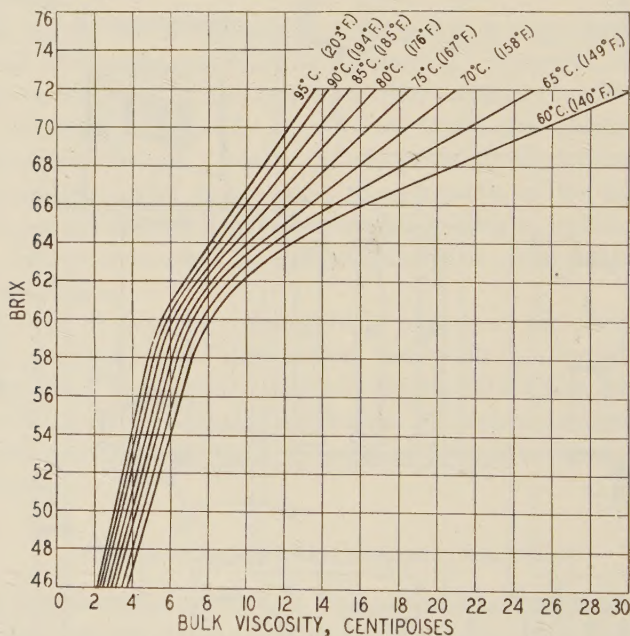


FIG. 2.

considered. The steam temperature was calculated from the steam pressure, and the heat input was determined by collecting the condensate from the steam side in calibrated receivers.

The viscosity of the molasses solutions used was determined with a MacMichael viscometer whose wire had been calibrated in terms of absolute viscosity on solutions whose viscosity was accurately known. From these readings Fig. 2 was prepared, giving the viscosity of the molasses used in this work over a range of temperatures and densities. The density of the molasses was recorded in degrees Brix. Where solutions other than molasses were used, the viscosity of a sample taken

at the end of the run was determined directly in the MacMichael viscometer at the apparent boiling point existing in that run.

In all experiments the velocity of the liquid at the entrance to the tube was held constant at about 9.4 ft. per second. This was accomplished by driving the Kinney circulation pump with a Reeves variable speed drive, and adjusting the drive frequently to hold the speed of the pump constant. The range covered in these experiments were as follows: Brix, 55-71°, apparent boiling point, 140-176° F., apparent overall temperature drop, 18-54° F.

TABLE I

Run No.	Brix Solution	t_s	t	Δt	u	t_f	Z_f	U
LK58....	55.42	193.98	175.15	18.83	9.38	184.56	4.80	318.3
LH49....	55.76	194.04	176.12	17.97	9.38	185.08	4.90	326.0
LG46....	55.16	174.11	155.79	18.32	9.38	164.45	5.35	285.4
LJ55....	55.67	176.04	157.98	18.06	9.38	167.01	5.55	297.2
LF43....	55.46	157.71	139.69	18.02	9.38	148.70	6.10	246.3
LI52....	55.52	158.66	139.53	19.13	9.38	149.09	6.10	246.5
LQ76....	60.25	194.65	176.13	18.52	9.38	185.39	6.30	238.7
LP73....	60.12	176.12	157.73	18.39	9.38	166.93	6.90	213.1
LO70....	60.36	158.32	139.06	19.26	9.38	148.69	7.90	183.8
LN67....	65.20	194.24	176.09	18.15	9.38	185.16	10.00	133.1
LM64....	65.10	176.03	158.00	18.03	9.38	167.01	11.30	120.6
LL61....	64.50	158.52	139.60	18.92	9.38	149.06	12.20	109.2
LT85....	71.22	194.02	176.04	17.98	9.38	185.03	14.75	87.3
LS82....	71.24	176.09	158.61	17.48	9.38	167.35	17.75	66.5
LR79....	71.44	158.14	139.68	18.46	9.38	148.91	24.05	40.0
LH50....	55.36	213.08	175.96	38.12	9.38	194.52	4.55	326.8
LK59....	55.52	212.29	175.30	36.99	9.38	193.79	4.55	338.1
LG47....	55.26	194.04	158.81	35.23	9.38	176.43	5.10	319.3
LJ56....	55.32	193.98	157.98	36.00	9.38	175.98	5.10	312.5
LF44....	55.06	174.11	140.11	34.00	9.38	157.11	5.65	261.1
LI53....	55.72	176.04	139.53	36.51	9.38	157.78	5.80	288.3
LQ77....	60.18	213.40	175.81	37.59	9.38	194.61	5.95	258.4
LP74....	60.15	194.02	157.91	36.11	9.38	175.96	6.60	231.7
LO71....	60.29	176.12	139.53	36.59	9.38	157.83	7.35	194.3
LN68....	65.26	213.37	176.50	36.87	9.38	194.94	9.50	140.9
LM65....	65.26	194.21	158.05	36.16	9.38	176.13	10.70	127.7
LL62....	64.52	176.03	139.28	36.75	9.38	157.65	11.50	114.4
LT86....	71.18	213.03	175.95	37.08	9.38	194.49	13.70	98.5
LS83....	71.30	194.00	158.40	35.60	9.38	176.10	16.15	72.1
LR80....	71.40	176.05	140.25	35.80	9.38	158.15	20.30	48.7
LH51....	55.42	229.98	176.12	53.84	9.38	203.05	4.25	339.8
LK60....	55.67	230.25	175.15	55.10	9.38	202.70	4.35	342.7
LJ57....	55.22	212.29	157.98	54.31	9.38	185.14	4.75	330.6
LG48....	55.46	213.28	158.54	56.74	9.38	185.91	4.80	327.2
LF45....	55.46	194.26	139.21	55.05	9.38	166.73	5.50	285.6
LI54....	55.97	193.98	140.16	53.82	9.38	166.59	5.50	288.3
LQ78....	60.15	230.13	176.11	54.02	9.38	203.12	5.70	264.8
LP75....	60.22	213.33	158.07	55.26	9.38	185.70	6.15	238.0
LO72....	60.17	194.16	139.73	54.43	9.38	166.95	6.95	198.7
LN69....	65.20	230.14	176.03	54.11	9.38	203.08	9.00	146.3
LM66....	65.10	212.83	157.91	54.92	9.38	185.37	9.90	127.7
LL63....	64.50	194.27	139.28	54.99	9.38	166.78	11.50	116.5
LT87....	71.25	230.05	176.09	53.96	9.38	203.07	13.15	103.0
LS84....	71.28	213.03	158.40	54.63	9.38	185.71	14.90	77.1
LR81....	71.29	194.00	140.61	53.39	9.38	167.31	17.90	50.8

DATA AND RESULTS

The important data are given in Table I. These were first worked up in three groups corresponding to the three temperature drops used. When $\log U^*$ was plotted against $\log Z$, the result was a straight line

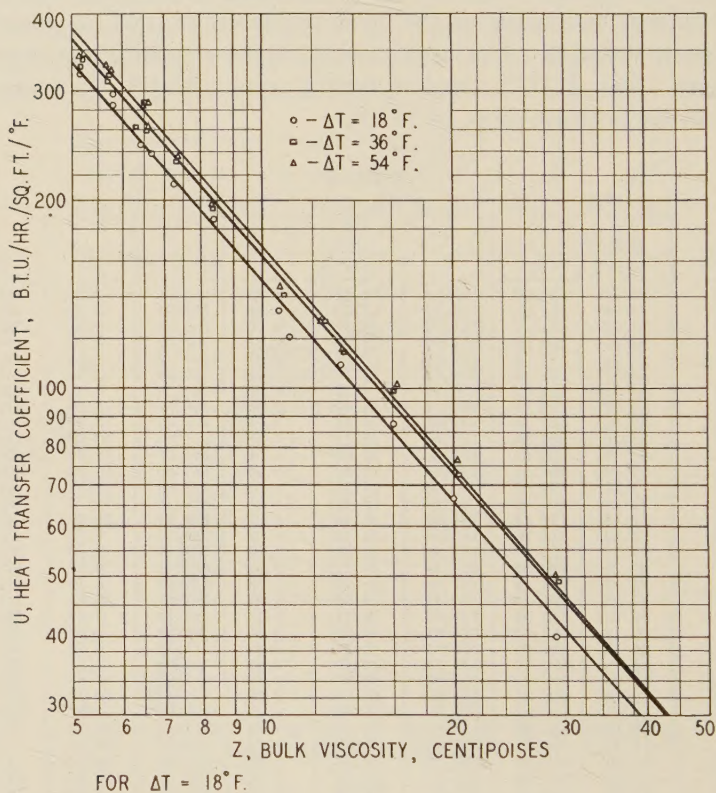


FIG. 3.

for each temperature drop, as shown in Fig. 3. Therefore, the first relationship is

$$U = \frac{K}{Z^a} \quad (1)$$

The values of K and a are as follows:

For $\Delta t = 18^\circ \text{ F.}$	$K = 2251$	$a = 1.18$
For $\Delta t = 36^\circ \text{ F.}$	$K = 2395$	$a = 1.17$
For $\Delta t = 54^\circ \text{ F.}$	$K = 2470$	$a = 1.17$

* See table of symbols at end of paper.

The values of a are constant within the experimental error and therefore the effect of variations in temperature drop is entirely contained in the constant K . To determine this relation, $\log K$ was plotted against $\log \Delta t$. This gives a straight line whose equation is:

$$K = 1770 \Delta t^{0.084} \quad (2)$$

Substituting this value of K , and 1.17 for a in Equation (1) gives,

$$U = \frac{1770 \Delta t^{0.084}}{Z^{1.17}} \quad (3)$$

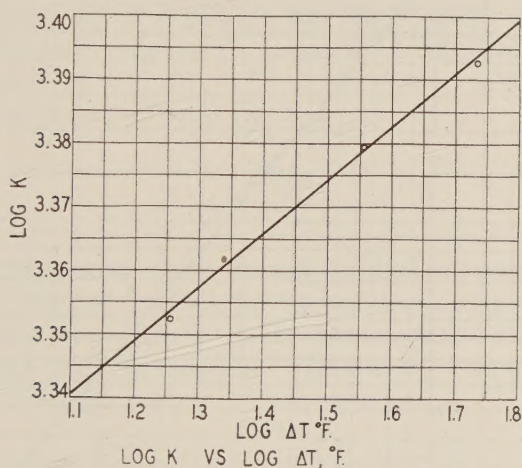


FIG. 4.

In Fig. 3 the curve for any one temperature drop contains points corresponding to runs made not only over a range of densities but also at various boiling points. If the change in boiling point had had any effect other than its effect on viscosity, the plots of Fig. 3 would not have been so constant. It can therefore be said that in the forced-circulation evaporator, the only effect of the boiling point of the liquid on heat-transfer coefficients is due to its effect on viscosity.

In a natural-circulation evaporator, changing the temperature drop changes the rate at which steam is evolved, and therefore greatly affects the velocity of circulation. In the forced-circulation evaporator there is no appreciable effect of this sort, and as Logan, Fragen, and Badger⁴ have shown, at velocities in the tube of 10 ft. per second or

over, there is little if any boiling in the tubes. Thus, the effect of Δt cannot be due to its effect on rate of circulation.

About the only effect of Δt that can easily be predicted is that changing the temperature drop changes the gradients through the films, and therefore changes the mean film temperature. This work ex-

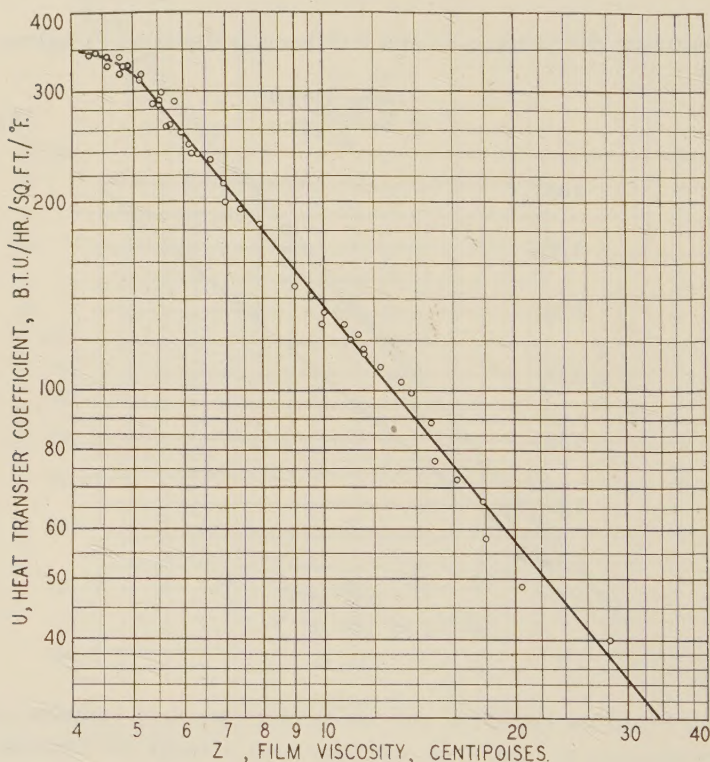


FIG. 5.

plicitly covers only those conditions where the steam film resistance is negligible; consequently it follows that the metal temperature is sensibly equal to steam temperature. If it is assumed that the mean film temperature is half-way between liquid temperature and metal temperature, it follows that t_f is half-way between the temperature of the liquid and the temperature of steam. It must be understood, however, that in all this work, the true mean temperature of the liquid has not

been measured, but instead of the mean bulk temperature of the liquid, the apparent boiling point has been used. The curves given by Logan, Fragen, and Badger⁴ show that there is a considerable difference in the two, but it should be remembered that throughout this paper, the intention is to define all quantities in terms of temperatures that are available to a designer. Consequently, t_f has been taken as half-way between the apparent boiling point and the steam temperature.

If viscosities at mean film temperature (Z_f) be used instead of viscosities at the apparent boiling point, (Z), and if $\log Z_f$ be plotted against $\log U$, the curve of Fig. 5 results. This gives a straight line, irrespective of boiling point and temperature drop, at all viscosities above 5 centipoises. It is possible that the deviation for viscosities below 5 centipoises represents the beginning of the effect of the steam film coefficients.

The equation for the line in Fig. 5 is

$$U = \frac{2354}{Z_f^{1.24}} \quad (4)$$

This equation seems to be general for molasses solutions over a considerable range of densities, boiling points and temperature drops.

A few runs were made on gelatin solutions in order to determine whether or not this relationship for molasses was general. The data are given in Table II.

TABLE II

Run No.	t_s	t	Δt	u	Z	U
LU88.....	175.71	138.79	36.92	9.38	11.43	110.0
LU89.....	175.64	139.12	36.52	9.38	11.72	103.3
LV90.....	175.66	138.92	36.74	9.38	5.82	233.4
LV91.....	175.71	138.92	36.79	9.38	5.97	221.1
LW92.....	175.69	138.79	36.90	9.38	3.55	425.1
LW93.....	175.69	138.92	36.77	9.38	3.44	426.6

A plot of $\log U$ against $\log Z$ for gelatin is shown in Fig. 6. The equation for this line is

$$U = \frac{1771}{Z^{1.15}} \quad (5)$$

The value of a is equal to the value obtained for molasses, within the experimental error. The value of K , however, differs considerably for the two substances. Since a general plot of viscosity vs. tem-

perature was not made for the gelatin, the values of Z_f cannot be calculated and hence the applicability of Equation (4) to gelatin could not be determined.

The constant K probably contains the effect of other specific properties of the solutions such as its thermal conductivity and its specific

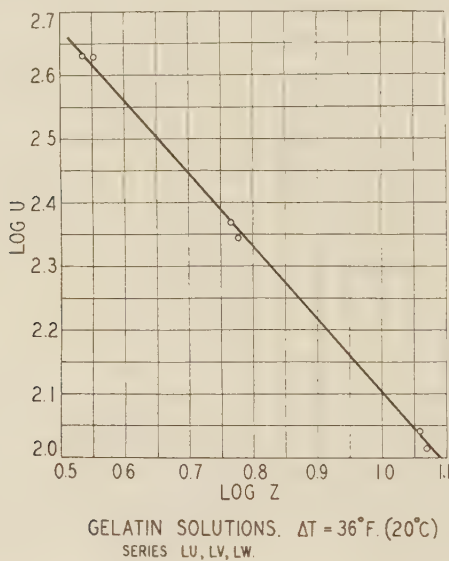


FIG. 6.

heat. Therefore, it is not surprising that the constants differ in the two cases. The effect of viscosity, however, is probably the same for both of these substances.

The tubes in which these experiments were run had been in service a considerable time and were appreciably roughened. Therefore, these equations will probably give results that are safe for use in practice.

SUMMARY

The effect of viscosity on the apparent overall heat-transfer coefficient in a forced-circulation evaporator, at constant velocity and constant temperature drop, is expressed by an equation of the form

$$U = \frac{K}{Z^a}$$

A large number of determinations gives a value of 1.17 for a for molasses. Six runs on gelatin give a value of 1.15 for a . It is probable that the value 1.17 is correct for both solutions. When film viscosity is defined as viscosity at a temperature half-way between apparent boiling point and steam temperature, molasses solutions give the equation

$$U = \frac{2354}{Z_f^{1.24}}$$

which is valid for all concentrations, boiling points and temperature drops covered in this investigation, provided that the viscosity is high enough so that the overall apparent coefficient is sensibly equal to the liquid-film coefficient. For these solutions this is true at all viscosities higher than 5 centipoises.

Although the constant K is different for different solutions and although it has not been proved that the exponent for a (1.17) is general for all solutions, the senior author has tried this type of equation on a number of materials in which a plot of the apparent coefficient against viscosity gives a straight line on log-log paper. These other cases do not involve experimental results accurate enough to warrant reproducing in this paper. However, the general relationship is extremely useful; as, for instance, when a substance is being concentrated to a certain density and it is desirable to know how the coefficients would change if the final density were changed. It would be more desirable, naturally, if the values of K could be predicted from some known property of solutions, but this is beyond the scope of the present investigation.

NOMENCLATURE

- a = constant.
- K = constant.
- t = apparent vapor temperature, degrees F.
- Δt = apparent temperature drop, degrees F.
- t_f = mean liquid film temperature, degrees F.
- t_s = apparent temperature of steam, degrees F.
- U = apparent overall heat-transfer coefficient, B.t.u. per sq. ft. per degree F. per hr.
- u = velocity of liquid at inlet of tubes, ft. per sec.
- Z = viscosity at t° F., centipoises.
- Z_f = viscosity at t_f° F., centipoises.

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